

ACTIVITY

Quadratic Polynomials

Objective

Learning geometrical representation of the factorization of the following quadratic polynomials:

1. $x^2 + 5x + 6$
2. $x^2 - 5x + 6$

Material Required

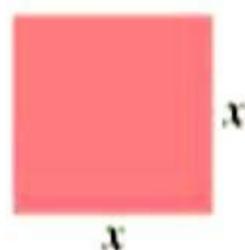
Glazed papers (blue, green, orange, yellow, and pink), white sheet of paper, geometry box, ruler, pair of scissors, and glue sticks.

Theory

1. Knowledge of quadratic equations.
2. Splitting of the middle term of a quadratic polynomial as $ax^2 + bx + c = a(x + p)(x - q)$, where,
 $-p + q = -\frac{b}{a}$, $-pq = \frac{c}{a}$
3. Area of a rectangle = $l \times b$
4. Area of a square = $(\text{side})^2$

Procedure

1. Every x^2 represents the area of pink-square of side x -units. Therefore, to represent $2x^2$, use two pink squares of side x units each. Take x as 3 units.



2. Every x represents the area of a green rectangular strip of dimensions $(1 \times x)$. [For $5x$, use 5 green strips each of dimensions $(1 \times x)$]

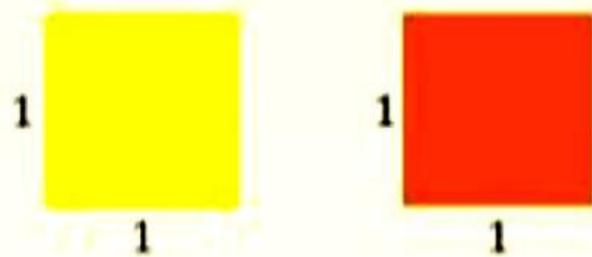


3. Every $(-x)$ is represented by a blue rectangular strip of dimensions $(1 \times x)$. For $(-5x)$, use 5 blue strips each of dimensions $(1 \times x)$.



4. All positive integers are represented by yellow unit squares and all negative integers are shown by

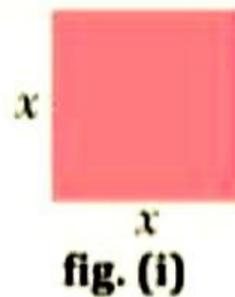
orange squares.



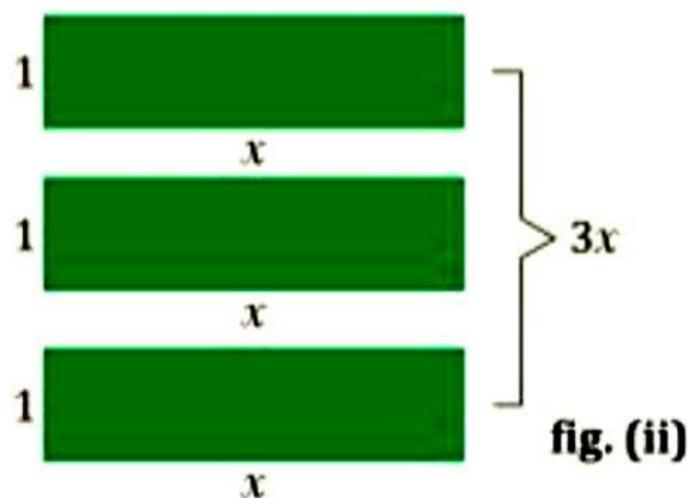
Case I:

Let us consider the expression $x^2 + 5x + 6$ which is of the form $(ax^2 + bx + c)$.

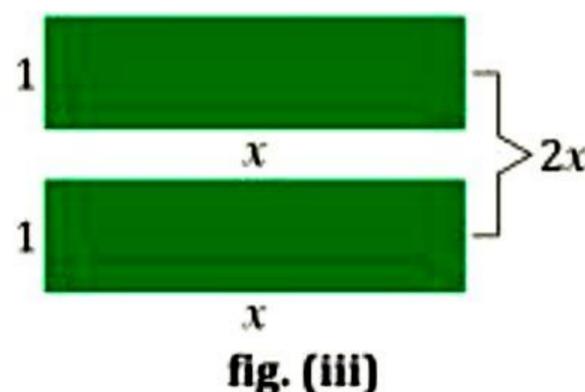
1. The polynomial $x^2 + 5x + 6 \Rightarrow x^2 + 3x + 2x + 6$ can be factorized as $(x + 3)(x + 2)$.
2. To present x^2 , draw a pink square of x units [fig. (i)].



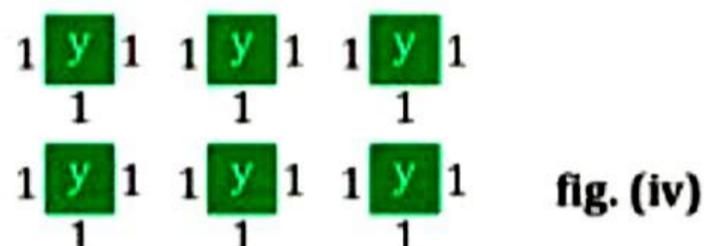
3. To represent $3x$, draw three rectangular strips of green color of dimension $(1 \times x)$ [fig. (ii)].



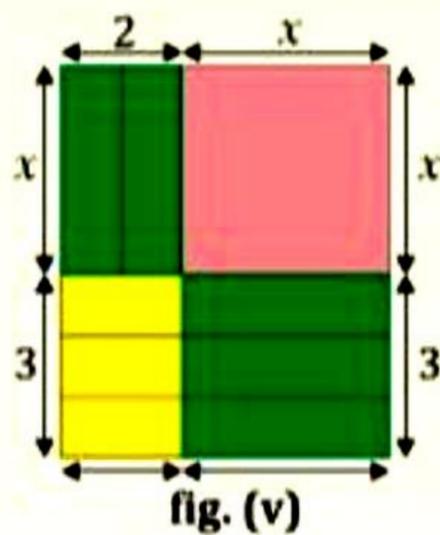
4. To represent $2x$, draw two green rectangular strips of dimensions $(1 \times x)$ [fig. (iii)].



5. To represent 6, draw 6 yellow unit squares [fig. (iv)].



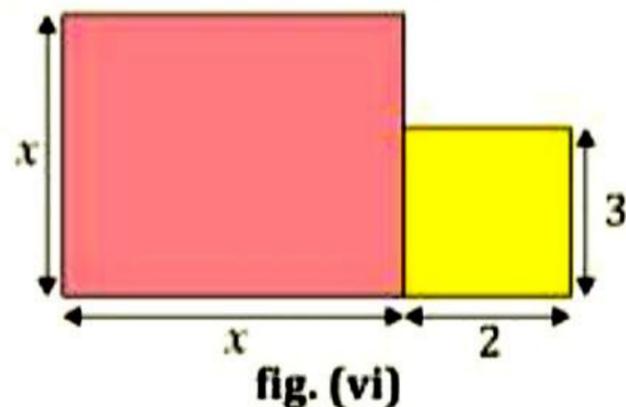
6. Cut all the strips from the glazed paper.
7. Now, paste all the strips together on the white sheet of paper as shown in fig. (v).



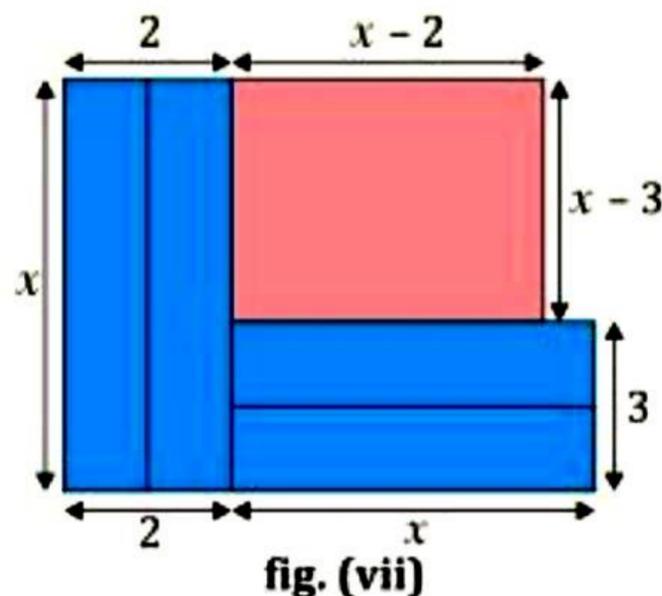
Case II:

Consider the expression $x^2 - 5x + 6$ and factorize it $x^2 - 3x - 2x + 6 = (x - 3)(x - 2)$.

1. Cut a pink square of dimension x units (say 8 units).
2. To represent 6, cut six yellow squares of dimension 1 unit.
3. To represent $-5x(-3x) + (-2x)$, cut five blue strips of dimension $(1 \times x)$.
4. Paste the pink square strips and all the yellow squares on a white sheet of paper as shown in fig. (vi).



5. Now, paste all the five blue strips over the pink polygon as shown in fig. (vii).



Observation and calculation

Case I:

$$x^2 + 5x + 6$$

Area of 5 green strips

$$= 5x = 2x + 3x$$

Area of pink square

$$= x^2$$

Area of 6 yellow unit squares

$$= 6$$

Total area of rectangle obtained

$$= x^2 + 5x + 6$$

$$= x^2 + 3x + 2x + 6$$

$$= (x + 3)(x + 2)$$

Case II:

$$x^2 - 5x + 6$$

$$\text{Area of 5 blue rectangular strips} = 5x \text{ (negative)}$$

$$\text{Area of a pink square} = x^2$$

$$\text{Area of 6 yellow unit squares} = 6$$

$$\text{Total area of pink rectangle} = (x - 2)(x - 3) = x^2 - 2x - 3x + 6$$

$$\text{obtained after pasting all strips} = x^2 - 5x + 6x$$

$$\therefore x^2 - 5x + 6 = (x - 3)(x - 2)$$

Result

We verified the factors of two quadratic polynomials geometrically by papercutting and pasting.

Learning Outcome

Above method gives us the geometrical interpretation of the factorization of quadratic expressions of the form

$$ax^2 + bx + c \text{ or } ax^2 - bx + c.$$

Remarks

1. Pasting blue strips over a pink area means reducing pink area.
2. The pink portion so obtained represents the factors of the given quadratic expression.
3. Students may take different color combinations.

Activity Time

By using paper cutting and pasting method, represent the factors of the following quadratic expressions:

1. $x^2 - x - 6$

2. $2x^2 + 5x + 2$

Viva Voce

Q1. How many linear factors can be in a quadratic polynomial?

Ans: 2 linear factors.

Q2. Find two numbers whose sum is 1 and the product is -12.

Ans: -3 and 4

Q3. Factorize: $x^2 + 7x + 12$.

Ans: $(x + 3)(x + 4)$

Q4. Find two numbers whose sum is 0 and the product is -6.

Ans: $\sqrt{6}$ and $-\sqrt{6}$

Q5. Is $y^2 + \frac{2}{y} + 5$ a polynomial?

Ans: No, as a power of y^2 in $\frac{2}{y}$ is -1.

Q6. Give one example of a binomial.

Ans: $(x + 5y)$.

Q7. Is $2 + x^2 + x$ a polynomial?

Ans: Yes.

Q8. Write the product of $(2x-1)(x+1)$.

Ans: $2x^2 + x - 1$

Q9. What is the degree of a quadratic polynomial?

Ans: The degree of a quadratic polynomial is 2

Multiple Choice Questions

Q 1. The expression $x^2 - 21x + 20$ is factorized as:

(a) $(x + 20)(x - 1)$

(b) $(x + 20)(x + 1)$

(c) $(x - 20)(x - 1)$

(d) $(x - 20)(x + 1)$

Q 2. $x^2 - 5x + 6$ is factorized as:

(a) $(x - 2)(x - 3)$

(b) $(x + 2)(x + 3)$

(c) $(x - 2)(x + 3)$

(d) $(x + 2)(x - 3)$

Q 3. One of the factors of $x^2 - 3x + 2$ is:

(a) $x + 2$

(b) $x + 1$

(c) $x - 1$

(d) $x + 3$

Q 4. The factorization of $3x^2 - 14x - 5$ by splitting the middle term is:

(a) $(x - 5)(3x + 1)$

(b) $(x + 5)(3x + 1)$

(c) $(x - 5)(3x - 1)$

(d) $(x + 5)(3x - 1)$

Q 5. If one zero of the quadratic polynomials $x^2 + 3x + k$ is 2, then the value of k is:

(a) 10

(b) -10

(c) 5

(d) -5

Q 6. $x^2 - 6x + 8$ is factorized as:

(a) $(x + 4)(x - 2)$

(b) $(x - 4)(x - 2)$

(c) $(x - 4)(x + 2)$

(d) $(x + 4)(x + 2)$

Q 7. $(x - 2)$ is a factor of:

(a) $x^2 - 7x - 18$

(b) $x^2 - 7x + 18$

(c) $x^2 + 7x + 18$

(d) $x^2 + 7x - 18$

Q 8. $(x^2 - 4x)(x^2 - 4x - 1) - 20$ is factorized as:

(a) $(x + 5)(x + 1)(x + 2)^2$

(b) $(x - 5)(x - 1)(x - 2)^2$

(c) $(x + 5)(x + 1)(x - 2)^2$

(d) $(x - 5)(x + 1)(x - 2)^2$

Q 9. The factorization of $84 - 2x - 2x^2$ is:

(a) $(x + 7)(12 - x)$

(b) $(x - 6)(14 + x)$

(c) $2(6 + x)(7 - x)$

(d) $2(6 - x)(7 + x)$

Q 10. Zeroes of the polynomial $x^2 + 4x - 5$ are:

(a) -1, 1

(b) 1, -5

(c) 0, -5

(d) -1, 5

ANSWER KEY

1.(c) 2.(a) 3.(c) 4.(a) 5.(b) 6.(b) 7.(d) 8.(d) 9.(d) 10.(b)